Name ________________________ ID3 _ _ _

ECS 452: Digital Communication Systems **2017/2**

HW 6 — Due: April 27, 4 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 5 pages.

(b) (1 pt) Work and write your answers **directly on these provided sheets** (not on other blank sheet(s) of paper). Hard-copies are distributed in class.

(c) (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of this page.

(d) (8 pt) Try to solve all non-optional problems.

(e) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Continue from the previous assignment. Consider a block code whose generator matrix is

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

(a) Suppose we receive $\underline{\mathbf{y}} = [1\,1\,1\,1\,0\,1]$.

(i) Minimum distance decoding:

i. Find the distance $d(\underline{\mathbf{x}}, \underline{\mathbf{y}})$ between this received vector $\underline{\mathbf{y}}$ and each of the possible codewords $\underline{\mathbf{x}}$.

$\underline{\mathbf{b}}$			$\underline{\mathbf{x}}$						$d(\underline{\mathbf{x}}, \underline{\mathbf{y}})$
0	0	0	0	0	0	0	0	0	
0	0	1	0	0	1	1	1	0	
0	1	0	0	1	0	0	1	1	
0	1	1	0	1	1	1	0	1	
1	0	0	1	0	0	1	0	1	
1	0	1	1	0	1	0	1	1	
1	1	0	1	1	0	1	1	0	
1	1	1	1	1	1	0	0	0	

ii. Use the answer in the previous part to find $\hat{\underline{\mathbf{x}}}$ and $\hat{\underline{\mathbf{b}}}$

(ii) Decoding via the syndrome:

i. Find the parity check matrix $\mathbf{H}$ of this code.

ii. Find the syndrome vector $\underline{\mathbf{s}}$.

iii. Use the answer in the previous parts to find $\hat{\underline{\mathbf{x}}}$ and $\hat{\underline{\mathbf{b}}}$

Problem 2. Consider a (15,11) Hamming code.

(a) Find the length of each codeword.

15

(b) For each codeword, how many bits are message bits?

11

(c) Suppose each codeword ~~the code~~ is constructed so that the parity bits are in the front and the message bits are in the back. Given an example of a generator matrix $\mathbf{G}$ and a corresponding parity check matrix $\mathbf{H}$ for such (15,11) Hamming code.

$n+1 = 16 \Rightarrow$ each colums of H will have $\log_2 16 = 4$ bits

$$G = [\,P \;\vdots\; \underset{11\times 11}{I}\,] \Rightarrow H = [\,I \;\vdots\; P^T\,] = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & \\ 0 & 1 & 0 & 0 & 1 & 0 & \\ 0 & 0 & 1 & 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 & \underbrace{0 \quad 0}_{\text{the remaining } 15-4=11 \text{ combinations.}} & & \cdots \end{array}\right]$$

Problem 3. Consider a convolutional encoder whose circuit diagram and a part of the corresponding state diagram is given in Figure 6.1. Write the suitable labels for the two arrows shown in the state diagram.

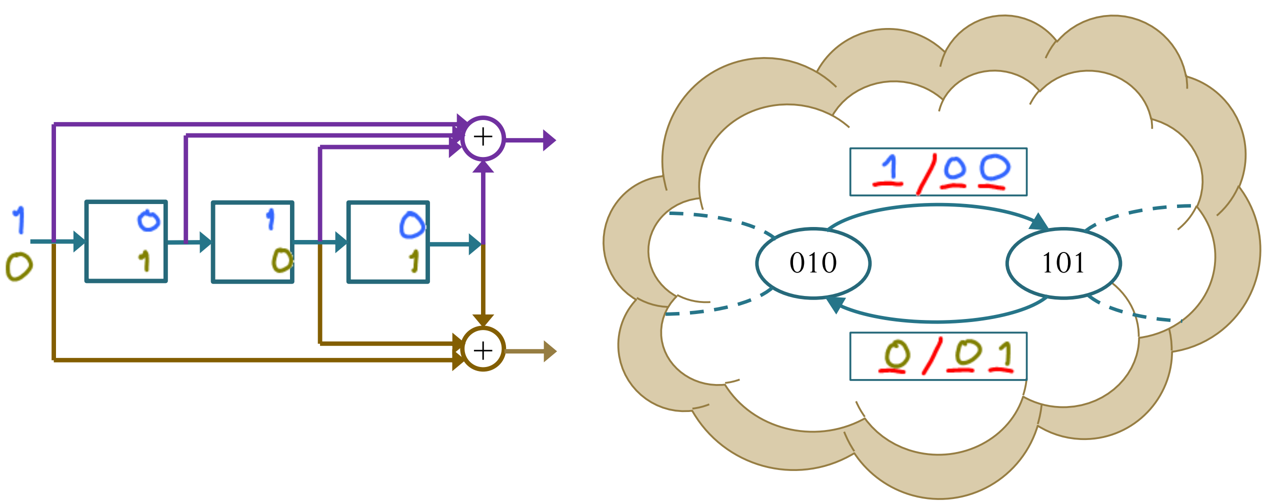

Figure 6.1: Circuit diagram and a part of the corresponding state diagram for a convolutional encoder. Only two states are shown in the state diagram.

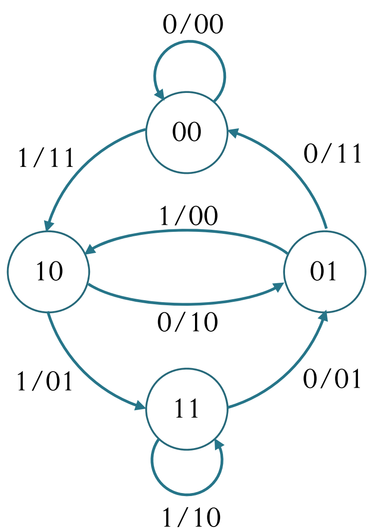

Figure 6.2: State diagram for a convolutional encoder

Problem 4. Consider a convolutional encoder whose state diagram is given in Figure 6.2.

(a) Find the code rate

(b) Suppose the data bits (message) are 0100101. Find the corresponding codeword.

(c) Find the data vector $\underline{\mathbf{b}}$ which gives the codeword $\underline{\mathbf{x}} = [001110111110110011]$

Extra Questions

Here are some optional questions for those who want more practice.

Problem 5. In the previous assignment, we consider the following encoding for a systematic linear block code:

- The bit positions that are powers of 2 (1, 2, 4, 8, 16, etc.) are check bits.
- The rest (3, 5, 6, 7, 9, etc.) are filled up with the k data bits.
- Each check bit forces the parity of some collection of bits, including itself, to be even.
  - To see which check bits the data bit in position i contributes to, rewrite i as a sum of powers of 2. A bit is checked by just those check bits occurring in its expansion.

For the case when the codeword's length $n = 7$, we found that

$$\mathbf{H} = \begin{array}{l} \text{LSB} \\ \\ \text{MSB} \end{array}\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

(a) Explain, from the elements inside the matrix $\mathbf{H}$, how this is a Hamming code.

1 2 3 4 5 6 7

(b) Consider the following decoding instruction:

- When a vector is observed, the receiver initializes a counter to zero. It then examines each check bit at position i ($i = 1, 2, 4, 8, \ldots$) to see if it gives the correct parity.
- If not, the receiver adds i to the counter. If the counter is zero after all the check bits have been examined (i.e., if they were all correct), the observed vector is accepted as a valid codeword. If the counter is nonzero, it contains the position of the incorrect bit.

Explain how the instruction above is the "same" as the decoding via the syndrome described in class.

Problem 6. Consider a block code whose generator matrix is

$$\mathbf{G} = \Big[\underbrace{\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{matrix}}_{P}\ \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I}\Big]. \Rightarrow H = [I \,\vdots\, P^T] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

A valid codeword was transmitted and potentially corrupted by the channel. Suppose that, at the decoder, the syndrome is found to be $\underline{\mathbf{s}} = [110]$. Find all the error patterns $\mathbf{e}$ that can cause this syndrome.

$$[1\,1\,0] = \underline{s} = \cancel{y}H^T = (\underline{x} \oplus \underline{e})H^T = \underbrace{\cancel{\underline{x}H^T}}_{0} \oplus \underline{e}H^T = [e_1\ e_2\ e_3\ e_4\ e_5\ e_6]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

So, we have three equations :

$$\left.\begin{aligned} e_1 \oplus e_4 \oplus e_6 &= 1 \\ e_2 \oplus e_4 \oplus e_5 &= 1 \\ e_3 \oplus e_5 \oplus e_6 &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} e_1 &= 1 \oplus e_4 \oplus e_6 \\ e_2 &= 1 \oplus e_4 \oplus e_5 \\ e_3 &= 0 \oplus e_5 \oplus e_6 \end{aligned}$$

e_1	e_2	e_3	e_4	e_5	e_6
1	1	0	0	0	0
0	1	1	0	0	1
1	0	1	0	1	0
0	0	0	0	1	1
0	0	0	1	0	0
1	0	1	1	0	1
0	1	1	1	1	0
1	1	0	1	1	1